BÀI TẬP GIỚI HẠN DÃY SỐ TỔNG HỢP

Câu 1: Tìm các giới hạn sau:

a). $\lim\frac{n-1}{n}$ b). $\lim\frac{n+2}{n+1}$ c). $\lim\frac{n^2-3n+5}{2n^2-1}$

d). $\lim\frac{3n^2+n-5}{2n^2+1}$ e). $\lim\frac{6n^3-2n+1}{2n^3-n}$ f). $\lim\frac{4n^4-n^2+1}{(2n+1)(3-n)(n^2+2)}$.

LỜI GIẢI

a) $\lim\frac{n-1}{n}=\lim\left(1-\frac{1}{n}\right)=0.$

b) $\lim\frac{n+2}{n+1}=\lim\frac{1+\frac{2}{n}}{1+\frac{1}{n}}=1.$ (Chia cả tử và mẫu cho n)

c) Chia cả tử và mẫu cho n^2 được:

$$\lim\frac{n^2-3n+5}{2n^2-1}=\lim\left(\frac{1-\frac{3n}{n^2}+\frac{5}{n^2}}{2-\frac{1}{n^2}}\right)=\lim\left(\frac{1-\frac{3}{n}+\frac{5}{n^2}}{2-\frac{1}{n^2}}\right)=\frac{1}{2}.$$

d) $\lim\frac{3n^2+n-5}{2n^2+1}=\lim\frac{3+\frac{n}{n^2}-\frac{5}{n^2}}{2+\frac{1}{n^2}}=\lim\frac{3+\frac{1}{n}-\frac{5}{n^2}}{2+\frac{1}{n^2}}=\frac{3}{2}.$

e) Chia cả tử và mẫu cho n^3 được:

$$\lim\frac{6n^3-2n+1}{2n^3-n}=\lim\frac{6-\frac{2n}{n^3}+\frac{1}{n^3}}{2-\frac{n}{n^3}}=\lim\left(\frac{6-\frac{2}{n^2}+\frac{1}{n^3}}{2-\frac{1}{n^2}}\right)=\frac{6}{2}=3.$$ f) $L=\lim\frac{4n^4-n^2+1}{(2n+1)(3-n)(n^2+2)}$

Ta có $4n^4-n^2+1=n^4\left(\frac{4n^4-n^2+1}{n^4}\right)=n^4\left(4-\frac{1}{n^2}+\frac{1}{n^4}\right)$; $2n+1=n\left(\frac{2n+1}{n}\right)=n\left(2+\frac{1}{n}\right)$;

$3-n=n\left(\frac{3-n}{n}\right)=n\left(\frac{3}{n}-1\right)$ và $n^2+2=n^2\left(\frac{n^2+2}{n^2}\right)=n^2\left(1+\frac{2}{n^2}\right)$

Từ đó ta có: $L=\lim\frac{4n^4-n^2+1}{n\left(2+\frac{1}{n}\right)n\left(\frac{3}{n}-1\right)n^2\left(1+\frac{2}{n^2}\right)}$

$$=\lim\frac{n^4\left(4-\frac{1}{n^2}+\frac{1}{n^4}\right)}{n^4\left(2+\frac{1}{n}\right)\left(\frac{3}{n}-1\right)\left(1+\frac{1}{n^2}\right)}=\lim\frac{4-\frac{1}{n^2}+\frac{1}{n^4}}{\left(2+\frac{1}{n}\right)\left(\frac{3}{n}-1\right)\left(1+\frac{1}{n^2}\right)}=\frac{4}{2.1}=2.$$

Câu 2: Tìm các giới hạn sau:

a). $\lim\frac{(n^2+2)(n-1)^2}{(n+1)(2n+3)^2}$ b). $\lim\frac{n^2+2\sqrt{n}+3}{2n^2+n-\sqrt{n}}$

c). $\lim \frac{2n^3-11n+1}{n^2-2}$　　d). $\lim \frac{\left(2n\sqrt{n}+1\right)\left(\sqrt{n}+3\right)}{(n+1)(n+2)}$

LỜI GIẢI

a). $\lim \frac{\left(n^2+2\right)(n-1)^2}{(n+1)(2n+3)^2}=\lim \frac{n^2\left(1+\frac{2}{n^2}\right)n^2\left(1-\frac{1}{n}\right)^2}{n\left(1+\frac{1}{n}\right)n^2\left(2+\frac{3}{n}\right)^2}=\lim \frac{\left(1+\frac{2}{n^2}\right)\left(1-\frac{1}{n}\right)^2}{\left(1+\frac{1}{n}\right)\left(2+\frac{3}{n}\right)^2}=\frac{1}{2}.$

b). $\lim \frac{1+\frac{2\sqrt{n}}{n^2}+\frac{3}{n^2}}{2+\frac{n}{n^2}-\frac{\sqrt{n}}{n^2}}=\lim \frac{1+\frac{2}{n\sqrt{n}}+\frac{3}{n^2}}{2+\frac{1}{n}-\frac{1}{n\sqrt{n}}}=\frac{1}{2}.$

c). $\lim \frac{\frac{2n^3}{n^2}-\frac{11n}{n^2}+\frac{1}{n^2}}{\frac{n^2}{n^2}-\frac{2}{n^2}}=\lim \frac{2n-\frac{11}{n}+\frac{1}{n^2}}{1-\frac{2}{n^2}}=\lim 2n=+\infty.$

d). $\lim \frac{\left(2n\sqrt{n}+1\right)\left(\sqrt{n}+3\right)}{(n+1)(n+2)}=\lim \frac{n\sqrt{n}\left(\frac{2n\sqrt{n}+1}{n\sqrt{n}}\right)\sqrt{n}\left(\frac{\sqrt{n}+3}{\sqrt{n}}\right)}{n\left(\frac{n+1}{n}\right)n\left(\frac{n+2}{n}\right)}$

$=\lim \frac{n\sqrt{n}\left(2+\frac{1}{n\sqrt{n}}\right)\sqrt{n}\left(1+\frac{3}{\sqrt{n}}\right)}{n\left(1+\frac{1}{n}\right)n\left(1+\frac{2}{n}\right)}=\lim \frac{\left(2+\frac{1}{n\sqrt{n}}\right)\left(1+\frac{3}{\sqrt{n}}\right)}{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}=\frac{2.1}{1.1}=2.$

Câu 3: Tìm các giới hạn sau:

a). $\lim \frac{\sqrt{9n^2-n+1}}{4n-2}$　　b). $\lim \frac{\sqrt{2n^4+3n-2}}{2n^2-n+3}$

c). $\lim \frac{\sqrt{2n+2}-\sqrt{n}}{\sqrt{n}}$　　d). $\lim \frac{\sqrt{3n^2+1}-\sqrt{n^2-1}}{n}$

LỜI GIẢI

a) $\lim \frac{\sqrt{9n^2-n+1}}{4n-2}=\lim \frac{\sqrt{n^2\left(9-\frac{1}{n}+\frac{1}{n^2}\right)}}{n\left(4-\frac{2}{n}\right)}=\lim \frac{n\sqrt{9-\frac{1}{n}+\frac{1}{n^2}}}{n\left(4-\frac{2}{n}\right)}=\lim \frac{\sqrt{9-\frac{1}{n}+\frac{1}{n^2}}}{4-\frac{2}{n}}=\frac{3}{2}.$

b). $\lim \frac{\sqrt{2n^4+3n-2}}{2n^2-n+3}=\lim \frac{\sqrt{n^4\left(2+\frac{3}{n^3}-\frac{2}{n^4}\right)}}{n^2\left(2-\frac{1}{n}+\frac{3}{n^2}\right)}$

$=\lim \frac{n^2\sqrt{2+\frac{3}{n^3}-\frac{2}{n^4}}}{n^2\left(2-\frac{1}{n}+\frac{3}{n^2}\right)}=\lim \frac{\sqrt{2+\frac{3}{n^3}-\frac{2}{n^4}}}{2-\frac{1}{n}+\frac{3}{n^2}}=\frac{\sqrt{2}}{2}.$

c). $\lim \frac{\sqrt{2n+2}-\sqrt{n}}{\sqrt{n}} = \lim \frac{\sqrt{n\left(2+\frac{2}{n}\right)}-\sqrt{n}}{\sqrt{n}} = \lim \frac{\sqrt{n}\sqrt{2+\frac{2}{n}}-\sqrt{n}}{\sqrt{n}} = \lim \frac{\sqrt{n}\left(\sqrt{2+\frac{2}{n}}-1\right)}{\sqrt{n}}$

$= \lim \left(\sqrt{2+\frac{2}{n}}-1\right) = \sqrt{2}-1.$

d). $\lim \frac{\sqrt{3n^2+1}-\sqrt{n^2-1}}{n} = \lim \frac{\sqrt{n^2\left(3+\frac{1}{n^2}\right)}-\sqrt{n^2\left(1-\frac{1}{n^2}\right)}}{n}$

$= \lim \frac{n\sqrt{3+\frac{1}{n^2}}-n\sqrt{1-\frac{1}{n^2}}}{n} = \lim \frac{n\left(\sqrt{3+\frac{1}{n^2}}-\sqrt{1-\frac{1}{n^2}}\right)}{n}$

$= \lim \left(\sqrt{3+\frac{1}{n^2}}-\sqrt{1-\frac{1}{n^2}}\right) = \sqrt{3}-1.$

Câu 4: Tìm các giới hạn sau:

a). $\lim \frac{3^n+5.4^n}{4^n+2^n}$ b). $\lim \frac{3^n-2.5^n}{7+3.5^n}$ c). $\lim \frac{2^n-3^n+5^{n+2}}{2^{n+1}+3^{n+2}+5^{n+1}}$ d). $\lim \frac{4.3^n+5^{n+1}}{3.2^n+5^n}$

LỜI GIẢI

a). $\lim \frac{3^n+5.4^n}{4^n+2^n} = \lim \frac{\frac{3^n}{4^n}+\frac{5.4^n}{4^n}}{\frac{4^n}{4^n}+\frac{2^n}{4^n}} = \lim \frac{\left(\frac{3}{4}\right)^n+5}{1+\left(\frac{2}{4}\right)^n} = \frac{5}{1} = 5.$

b). $\lim \frac{3^n-2.5^n}{7+3.5^n} = \lim \frac{\frac{3^n}{5^n}-\frac{2.5^n}{5^n}}{\frac{7}{5^n}+\frac{3.5^n}{5^n}} = \lim \frac{\left(\frac{3}{5}\right)^n-2}{\frac{7}{5^n}+3} = -\frac{2}{3}.$

c). $\lim \frac{2^n-3^n+5^{n+2}}{2^{n+1}+3^{n+2}+5^{n+1}} = \lim \frac{2^n-3^n+5^2.5^n}{2.2^n+3^2.3^n+5.5^n}$

$= \lim \frac{\frac{2^n}{5^n}-\frac{3^n}{5^n}+\frac{5^2.5^n}{5^n}}{\frac{2.2^n}{5^n}+\frac{3^2.3^n}{5^n}+\frac{5.5^n}{5^n}} = \lim \frac{\left(\frac{2}{5}\right)^n-\left(\frac{3}{5}\right)^n+25}{2.\left(\frac{2}{5}\right)^n+9.\left(\frac{3}{5}\right)^n+5} = 5.$

d). $\lim \frac{4.3^n+5^{n+1}}{3.2^n+5^n} = \lim \frac{4.3^n+5.5^n}{3.2^n+5^n} = \lim \frac{\frac{4.3^n}{5^n}+\frac{5.5^n}{5^n}}{\frac{3.2^n}{5^n}+\frac{5^n}{5^n}} = \lim \frac{4.\left(\frac{3}{5}\right)^n+5}{3.\left(\frac{2}{5}\right)^n+1} = 5.$

Câu 5: Tìm các giới hạn sau:

a). $\lim \frac{2^n+(-5)^n}{2.3^n+3.(-5)^n}$ b). $\lim \frac{\sqrt{9^n+1}}{3^n-1}$ c). $\lim \frac{(-1)^n.2^{5n+1}}{3^{5n+2}}$ d). $\lim \frac{n+\sqrt{n^2+1}}{n.3^n}$

LỜI GIẢI

a). $\lim \frac{2^n+(-5)^n}{2.3^n+3.(-5)^n}=\lim \frac{\frac{2^n}{(-5)^n}+\frac{(-5)^n}{(-5)^n}}{\frac{2.3^n}{(-5)^n}+\frac{3.(-5)^n}{(-5)^n}}=\lim \frac{\left(\frac{-2}{5}\right)^n+1}{2.\left(\frac{-3}{5}\right)^n+3}=\frac{1}{3}.$

b). $\lim \frac{\sqrt{9^n+1}}{3^n-1}=\lim \frac{\frac{\sqrt{9^n-1}}{3^n}}{\frac{3^n-1}{3^n}}=\lim \frac{\sqrt{1+\frac{1}{9^n}}}{1-\frac{1}{3^n}}=1.$

c). $\lim \frac{(-1)^n.2^{5n+1}}{3^{5n+2}}=\lim \frac{(-1)^n.2.2^{5n}}{3^2.3^{5n}}=\lim \frac{(-1).2}{9}.\left(\frac{2}{3}\right)^{5n}=0.$

d). $L=\lim \frac{n+\sqrt{n^2+1}}{n.3^n}=\lim \frac{\frac{n+\sqrt{n^2+1}}{n}}{\frac{n.3^n}{n}}=\lim \frac{1+\sqrt{1+\frac{1}{n^2}}}{3^n}=\lim \frac{1}{3^n}\left(1+\sqrt{1+\frac{1}{n^2}}\right)$. Có $\lim \frac{1}{n^2}=0$ nên

$\lim \left(1+\sqrt{1+\frac{1}{n^n}}\right)=2$ và $\lim \frac{1}{3^n}=0$. Do đó $L=0$.